Universitat de Lleida

DEGREE CURRICULUM

# LINEAR ALGEBRA

Coordination: MESSEGUE BUISAN, ARNAU

Academic year 2023-24

# LINEAR ALGEBRA 2023-24

## Subject's general information

| | |
|---|---|
| **Subject name** | LINEAR ALGEBRA |
| **Code** | 102320 |
| **Semester** | 2nd Q(SEMESTER) CONTINUED EVALUATION |
| **Typology** | <table><tr><th>Degree</th><th>Course</th><th>Character</th><th>Modality</th></tr><tr><td>Bachelor's degree in Industrial Organization and Logistics Engineering</td><td>1</td><td>COMMON/CORE</td><td>Attendance-based</td></tr><tr><td>Common branch in industrial engineering programs - Igualada</td><td>1</td><td>COMMON/CORE</td><td>Attendance-based</td></tr><tr><td>Doble titulació: Grau en Enginyeria en Organització Industrial i Logística i Grau en Administració i Direcció d'Empreses</td><td>1</td><td>COMMON/CORE</td><td>Attendance-based</td></tr><tr><td>Not informed</td><td>1</td><td>COMMON/CORE</td><td>Attendance-based</td></tr></table> |
| **Course number of credits (ECTS)** | 6 |
| **Type of activity, credits, and groups** | <table><tr><th>Activity type</th><th>PRAULA</th><th>TEORIA</th></tr><tr><td>Number of credits</td><td>3</td><td>3</td></tr><tr><td>Number of groups</td><td>2</td><td>1</td></tr></table> |
| **Coordination** | MESSEGUE BUISAN, ARNAU |
| **Department** | MATHEMATICS |
| **Teaching load distribution between lectures and independent student work** | 1.5 h of student workload for each hour of face-to-face lessons |
| **Important information on data processing** | Consult this link for more information. |
| **Language** | Catalan. |
| **Distribution of credits** | 3 theoretical credits and 3 practical credits. |

| Teaching staff | E-mail addresses | Credits taught by teacher | Office and hour of attention |
|---|---|---|---|
| MESSEGUE BUISAN, ARNAU | arnau.messegue@udl.cat | 9 | |

## Subject's extra information

This course requires continuous work throughout the semester in order to achieve the objectives set. It requires critical thinking and capacity for abstraction.

It is recommended to visit frequently the Virtual Campus webpage associated to the course, since all the corresponding information will be announced there.

## Learning objectives

**Learning objectives:**

1. Discuss and solve linear systems of equations.
2. Describe vector spaces and subspaces.
3. Perform subspace operations: intersection, sum and direct sum.
4. Describe linear aplications from their kernel and their image. Perfom changes of basis in a linear aplication.
5. Determine whether or not an endomorphism diagonalizes.
6. Apply the diagonalization of endomorphisms to determine powers of matrices, to solve linear recurrences, and differential systems of linear equations.
7. Classify a bilinear form. Express the quadratic form associate as a sum of squares.
8. Apply the scalar product of vectors to calculate distances and angles.
9. Apply the orthonormalization Gram-Schmidt process in order to compute orthonormal basis.
10. Learn to apply the theoretical items in the resolution of different questions of engineering.

## Competences

B01. That the students have demonstrated to have and understand connections in an area of study that starts from the base of general secondary education and it is usually found at a level that, although it is supported by advanced textbooks, also includes some aspects that involve knowledge coming from the forefront of their field of study.

B02. That students know how to apply their knowledge to their work or vocation in a professional way and possess the competencies that are usually demonstrated through the elaboration and defense of arguments and the resolution of problems within their area of study.

CG3. Synthesize basic and technological subjects, which enables them to learn new methods and theories, and gives them the versatility to adapt to new situations.

CG4. Solve problems with initiative, make decisions, creativity, critical reasoning, and communicate and transmit knowledge, abilities, and skills in the field of Chemical/ Industrial Organization and Logistic Engineering.

CG10. Work in a multilingual and multidisciplinary environment.

CE1. Develop the ability to solve mathematical problems that may arise in engineering. Ability to apply knowledge about: linear algebra; geometry; differential geometry; differential and integral calculus; differential equations and partial derivatives; numerical methods; numerical algorithmic; statistics and optimization.

CT5. Acquire essential notions of scientific thought.

## Subject contents

1. Matrices, determinats and systems of linear equations.

1. Types of matrices and operations with matrices.
2. Invertible matrices.
3. Determinant of a square matrix of order n. Cases n=2, n=3 and n>3.
4. Rank of a matrix.
5. Elementary transformations by rows. Gauss elimination method.
6. Expression of a system of linear equations by matrices.
7. Rouché-Frobenius theorem.
8. Cramer Systems.
9. Applications: Adjustment of chemical reactions. Population dynamics. Cryptography.

2. **Vector spaces.**

1. Definition, properties and examples.
2. Linear combination: linear dependence and linear independence.
3. Generating system.
4. Basis of a vector space: Definition, dimensions, components.
5. Vector Subspace. Linear varieties.
    - Description by using generators and equations.
    - Gauss Transformations.
    - Basis and dimension.
6. Changes of basis. Basis change matrix.
7. Operations on subspaces.
    - Containment and equality.
    - Intersection and sum.
    - Grassman formula.
    - Complementary and direct sum.
8. Applications: Color.

3. **Linear transformations and endomorphism diagonalization.**

1. Definition and properties of a linear transformation.
2. Determination by the image in a basis.
3. Kernel and image of a linear transformation.
4. Matrix representation of a linear transformation.
5. Changes of basis in a linear transformation. Similar matrices.
6. Operations with linear applications.
    - Sum of applications.
    - Product by a scalar.
    - Composition of applications.

7. Applications: Image processing.

4. **Endomorphism diagonalization.**

1. Diagonal matrix: eigenvalues and eigenvectors.
2. Characteristic polynomial of an endomorphism: definition and calculation.
3. Cayley-Hamilton theorem.
4. Algebraic multiplicity.
5. Subspace generated by an eigenvector: geometric multiplicity.
6. Characterization of diagonalizable endomorphisms.
7. Applications of diagonalization: Powers of matrices, Linear Recurrences and Systems of linear differential equations. Internet search engines: The Pagerank.

5. **Bilinear forms and quadratic forms.**

1. Properties of the ordinary scalar product in $R^n$. Concept of bilinear form.
2. Matrix representation of a bilinear form in a basis. Concept of orthogonality.
3. Symmetric bilinear form. Gauss method to find an orthogonal basis. Gram-Schmidt process of orthonormalization.
4. Spectral decomposition.
5. Applications of orthogonality: ordinary least squares (OLS) or linear least squares.
6. Definition of rank and signature. Effective methods of calculation of these invariants.
7. Scalar Products. Definition of the associated norms. Orthonormal basis. Applications to geometry.
8. Quadratic form associated to a bilinear form.
9. Applications: Expression of a quadratic form as a linear combination of squares.
10. Classification of conics and quadrics.

## Methodology

The activities are divided into two types that complement each other: lecture sessions and problem-solving sessions.

- **Lecture sessions:** In the lectures, we introduce concepts and relevant theoretical results and illustrate them with examples and exercises. We will use blackboard and computer resources.
- **Problem-solving sessions:** These sessions are devoted to exercises in order to gradually consolidate the concepts and ideas developed in the lecture sessions. The students will work these exercises individually or as a team. In certain problems, some computer tools can be used to solve them.

In addition, students will have the responsibility to reinforce their knowledge autonomously based on the teaching material provided or recommended by the teacher.

## Development plan

**Activities:**

1. **Lectures:** The issues described in the section Contents will be held in the master classes throughout the different weeks of lessons. From the begining until the end of the semester. **Classroom hours** 16h **Personal learning hours** 24h
2. **Resolution of problems:** The issues described in section Contents will be developed in practices in the classroom during the teaching weeks. From the begining until the end of the semester. **Classroom hours** 16h **Personal learning hours** 24h
3. **Tutorial:** Exam reviews and consulting hours will be used to answer questions and clarify concepts. From the begining until the end of the semester.
4. **Evaluation:** The evaluation, described in the section Evaluation, will be developed by written in-person tests (exams) and the delivery of exercises. From the begining until the end of the semester. **Classroom**

**hours** 4h (or 6h).

Working plan:

| Semana | Metodología | Contenidos | Horas presenciales | Horas no presenciales |
|---|---|---|---|---|
| 1-8 | Lectures | 1, 2, 3 | 16 | 24 |
| 1-8 | Resolution of problems | 1, 2, 3 | 16 | 24 |
| 9 | Evaluation. Partial Exam 1 | 1, 2, 3 | 2 | |
| 10-15 | Lectures | 4, 5 | 10 | 15 |
| 10-15 | Resolution of problems | 4, 5 | 10 | 15 |
| 16-17 | Evaluation. Partial Exam 2 | 4, 5 | 2 | |
| 19 | Reevaluation. Exam | 1, 2, 3 and/or 4,5 | 2 | |

## Evaluation

- Control Exam 1 (C1): 10%.

- Partial Exam 1 (P1): 40%.

- Control Exam 2 (C2):: 10%.

- Partial Exam 2 (P2):: 40%.

**Final Mark = 0.1·C1 + 0.4·P1 + 0.1·C2 + 0.4·P2**

To compute the final mark, there is no minimum mark for the control exams, the partial exams have 2.5 as a minimum mark (over 10).

If necessary, the students can resit Partial Exams 1 and 2 (80%). The final mark will be computed after these resits.

It is mandatory to do both partial exams (to attend and develop them).

All the exams can be exclusively done with a non-programable calculator.

## Bibliography

**BASIC BIBLIOGRAPHY**

1. **Isaac A. García, Jaume Giné, Problemas resueltos de Álgebra Lineal. Col.lecció Eines, no. 45. Ed. de la UdL. Lleida, 2003. ALSO IN E-BOOK!**

**COMPLEMENTARY BIBLIOGRAPHY**

1. **Joan Gimbert, Xavier Hernández, Nacho López, Josep Maria Miret, Ramiro Moreno, Magda Valls. Curs pràctic d'àlgebra per a informàtics. Col.lecció Eines, no. 48. Ed. de la UdL. Lleida, 2004. ALSO IN E-BOOK!**
2. Nina Bijedic; Joan Gimbert; Josep M. Miret; Magda Valls. Elements of Discrete Mathematical Structures for

ComputerScience. Univerzittska knjiga Mostar, 2007.Kenneth ROSEN, Matemática Discreta y sus aplicaciones. McGraw-Hill Interamericana, 5a. edició, 2006.
3. Howard ANTON. Introducción al Álgebra Lineal. Ed. Limusa, 3a. edició, 1990.