



Universitat de Lleida

DEGREE CURRICULUM
LINEAR ALGEBRA

Coordination: DALFÓ SIMÓ, CRISTINA

Academic year 2019-20

Subject's general information

Subject name	LINEAR ALGEBRA		
Code	102320		
Semester	2nd Q(SEMESTER) CONTINUED EVALUATION		
Typology	Degree	Course	Character
	Bachelor's degree in Industrial Organization and Logistics Engineering	1	COMMON
	Not informed	1	COMMON
Modality	Attendance-based		
Course number of credits (ECTS)	6		
Type of activity, credits, and groups	Activity type	PRAULA	TEORIA
	Number of credits	3	3
	Number of groups	2	2
Coordination	DALFÓ SIMÓ, CRISTINA		
Department	MATHEMATICS		
Teaching load distribution between lectures and independent student work	60 classroom hours and 90 hours of independent work.		
Important information on data processing	Consult this link for more information.		
Language	Catalan.		
Distribution of credits	3 theoretical credits and 3 practical credits.		

Teaching staff	E-mail addresses	Credits taught by teacher	Office and hour of attention
DALFÓ SIMÓ, CRISTINA	cristina.dalfo@udl.cat	12	

Subject's extra information

This course requires continuous work throughout the semester in order to achieve the objectives set. It requires critical thinking and capacity for abstraction.

It is recommended to visit frequently the Virtual Campus webpage associated to the course, since all the corresponding information will be announced there.

Learning objectives

Learning objectives:

1. Discuss and solve linear systems of equations.
2. Describe vector spaces and subspaces.
3. Perform subspace operations: intersection, sum and direct sum.
4. Describe linear applications from their kernel and their image. Perform changes of basis in a linear application.
5. Determine whether or not an endomorphism diagonalizes.
6. Apply the diagonalization of endomorphisms to determine powers of matrices, to solve linear recurrences, and differential systems of linear equations.
7. Classify a bilinear form. Express the quadratic form associate as a sum of squares.
8. Apply the scalar product of vectors to calculate distances and angles.
9. Apply the orthonormalization Gram-Schmidt process in order to compute orthonormal basis.
10. Learn to apply the theoretical items in the resolution of different questions of engineering.

Competences

Basic competences

B01 That students have demonstrated to possess and understand knowledge in an area of study that starts from the base of general secondary education, and it is usually found at a level that, although supported by advanced textbooks, also includes some aspects that imply knowledge coming from the vanguard of his/her field of study.

B02 That students know how to apply their knowledge to their work or vocation in a professional manner and they possess the skills that are usually proved through the elaboration and defense of arguments and the resolution of problems within their area of study.

Transversal competences

CT5. To apply essential notions of scientific thinking.

General competences

CG3. To synthesize basic and technological subjects, which enable them to learn new methods and theories, and provide them with versatility to adapt to new situations.

CG4. To solve problems with initiative, make decisions, creativity, critical reasoning and to communicate and transmit knowledge, skills and abilities in the field of Chemical Engineering/Industrial Organization Engineering.

CG10. To work in a multilingual and multidisciplinary environment.

CG11. To understand and apply the necessary legislation in the exercise of the profession of Chemical Engineering/Industrial Organization Engineering

Specific competences

CE1. To develop the ability to solve mathematical problems in the engineering field. Aptitude to apply knowledge on: linear algebra; geometry; differential geometry; differential and integral calculus; differential equations and in partial derivatives; numerical methods; algorithmic, numerical; statistics and optimization.

Subject contents

1. Matrices, determinants and systems of linear equations.

1. Types of matrices and operations with matrices.
2. Invertible matrices.
3. Determinant of a square matrix of order n . Cases $n=2$, $n=3$ and $n>3$.
4. Rank of a matrix.
5. Elementary transformations by rows. Gauss elimination method.
6. Expression of a system of linear equations by matrices.
7. Rouché-Frobenius theorem.
8. Cramer Systems.
9. Applications: Adjustment of chemical reactions. Population dynamics. Cryptography.

2. Vector spaces.

1. Definition, properties and examples.
2. Linear combination: linear dependence and linear independence.
3. Generating system.
4. Basis of a vector space: Definition, dimensions, components.
5. Vector Subspace. Linear varieties.
 - Description by using generators and equations.
 - Gauss Transformations.
 - Basis and dimension.
6. Changes of basis. Basis change matrix.
7. Operations on subspaces.
 - Containment and equality.
 - Intersection and sum.
 - Grassman formula.
 - Complementary and direct sum.
8. Applications: Color.

3. Linear transformations and endomorphism diagonalization.

1. Definition and properties of a linear transformation.
2. Determination by the image in a basis.
3. Kernel and image of a linear transformation.
4. Matrix representation of a linear transformation.
5. Changes of basis in a linear transformation. Similar matrices.
6. Operations with linear applications.
 - Sum of applications.
 - Product by a scalar.
 - Composition of applications.

7. Applications: Image processing.

4. Endomorphism diagonalization.

1. Diagonal matrix: eigenvalues and eigenvectors.
2. Characteristic polynomial of an endomorphism: definition and calculation.
3. Cayley-Hamilton theorem.
4. Algebraic multiplicity.
5. Subspace generated by an eigenvector: geometric multiplicity.
6. Characterization of diagonalizable endomorphisms.
7. Applications of diagonalization: Powers of matrices, Linear Recurrences and Systems of linear differential equations. Internet search engines: The Pagerank.

5. Bilinear forms and quadratic forms.

1. Properties of the ordinary scalar product in \mathbb{R}^n . Concept of bilinear form.
2. Matrix representation of a bilinear form in a basis. Concept of orthogonality.
3. Symmetric bilinear form. Gauss method to find an orthogonal basis. Gram-Schmidt process of orthonormalization.
4. Spectral decomposition.
5. Applications of orthogonality: ordinary least squares (OLS) or linear least squares.
6. Definition of rank and signature. Effective methods of calculation of these invariants.
7. Scalar Products. Definition of the associated norms. Orthonormal basis. Applications to geometry.
8. Quadratic form associated to a bilinear form.
9. Applications: Expression of a quadratic form as a linear combination of squares.
10. Classification of conics and quadrics.

Methodology

The activities are divided into two types that complement each other: lecture sessions and problem solving sessions.

- **Lecture sessions:** In the lectures we introduce concepts and relevant theoretical results and illustrate them with examples and exercises. We will use blackboard and computer resources.
- **Problem solving:** These sessions are devoted to exercises in order to gradually consolidate the concepts and ideas developed in the lecture sessions. The students will work these exercises individually or as a team. In certain problems, some computer tools can be used to solve them.

In addition, students will have the responsibility to reinforce their knowledge autonomously based on the teaching material provided or recommended by the teacher.

Development plan

Activities:

1. **Lectures:** The issues described in the section Contents will be held in the master classes throughout the different weeks of lessons. From the beginning until the end of the semester. **Classroom hours** 16h **Personal learning hours** 24h
2. **Resolution of problems:** The issues described in section Contents will be developed in practices in the classroom during the teaching weeks. From the beginning until the end of the semester. **Classroom hours** 16h **Personal learning hours** 24h
3. **Tutorial:** Exam reviews and consulting hours will be used to answer questions and clarify concepts. From the beginning until the end of the semester.
4. **Evaluation:** The evaluation, described in the section Evaluation, will be developed by written in-person tests (exams) and the delivery of exercises. From the beginning until the end of the semester. **Classroom hours** 4h (or 6h).

Working plan:

Semana	Metodología	Contenidos	Horas presenciales	Horas no presenciales
1-8	Lectures	1, 2, 3	16	24
1-8	Resolution of problems	1, 2, 3	16	24
9	Evaluation. Partial Exam 1	1, 2, 3	2	
10-15	Lectures	4, 5	10	15
10-15	Resolution of problems	4, 5	10	15
16-17	Evaluation. Partial Exam 2	4, 5	2	
19	Reevaluation. Exam	1, 2, 3 and/or 4,5	2	

Evaluation

- Control Exam 1: 10%.
- Partial Exam 1: 40%.
- Control exam 2: 10%.
- Partial Exam 2: 40%.

To pass this subject, there is no minimum mark for the control exams, the partial exams have 2.5 as a minimum mark (over 10).

If necessary, the students can make the reevaluation of Partial Exams 1 and 2 (80%).

It is mandatory to do to all the four exams (to attend and develop them).

All the exams can be exclusively done with a non-programable calculator.

All the exams can be done with the paper with formulas given by the teacher.

Bibliography

BASIC BIBLIOGRAPHY

1. H. Anton, Introducción al Álgebra Lineal., Ed. Limusa, México, 1997.
2. I. A. García, J. Giné, Problemas resueltos de Álgebra Lineal. Col.lecció Eines, no.45. Ed. de la UdL. Lleida. 2003.

REVIEWING BIBLIOGRAPHY

1. J. Sorolla Bardají, Introducció a la Matemàtica, ISBN: 978-84-616-4854-2

RECOMMENDED BIBLIOGRAPHY

1. M. Alsina, J. M. Miret, A. Rio, Càlcul i Àlgebra. Resums i Problemes (amb solucions), 1991.
2. J. Arvesú, F. Marcellán, J. Sánchez, Problemas resueltos de álgebra lineal, Madrid, International Thomson, 2005.
3. C. Alsina, E. Trillas, Lecciones de Álgebra y Geometría. Editorial Gustavo Gili, S.A., Barcelona, 1984.
4. M. Castellet, I. Llerena, Àlgebra lineal i geometria., Publicacions de la UAB, Barcelona, 1988.
5. P. M. Cohn, Elements of linear algebra, Chapman and Hall, London, 1994.
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7. M. T. García González, A. Ruiz Oliarria, M. M. Saiz Jarabo, Álgebra. Teoría y ejercicios. Ed. Paraninfo, Madrid, 1993.
8. **J. Gimbert, X. Hernández, N. López, J. M. Miret, R. Moreno, M. Valls. Curs pràctic d'álgebra per a informàtics. Col.lecció Eines, no. 48. Ed. de la UdL. Lleida, 2004.**
9. S. Lang, Introducción al Álgebra Lineal. Ed. Addison-Wesley, Wilmington, Delaware, 1990.
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11. J. Rojo, I. Martín, Ejercicios y problemas de Álgebra Lineal, Ed. Vector Ediciones, Madrid, 1989.