



Universitat de Lleida

# DEGREE CURRICULUM

# **NUMERICAL METHODS**

Academic year 2014-15

## Subject's general information

<b>Subject name</b>	Numerical Methods
<b>Code</b>	102102
<b>Semester</b>	1st S Continuous Assessment
<b>Typology</b>	Face to face
<b>ECTS credits</b>	6
<b>Theoretical credits</b>	3
<b>Practical credits</b>	3
<b>Office and hour of attention</b>	Schedule: to agree with the student. Location: Office of the professor.
<b>Department</b>	Mathematics
<b>Teaching load distribution between lectures and independent student work</b>	40% classroom 60% homework
<b>Modality</b>	Presencial
<b>Important information on data processing</b>	Consult <a href="#">this link</a> for more information.
<b>Language</b>	Language Percentage of use Català 50.0 Anglès 0.0 Castellà 50.0
<b>Degree</b>	Degree in Mechanical Engineering. Degree in Automation and Industrial Electronic Engineering
<b>Office and hour of attention</b>	Schedule: to agree with the student. Location: Office of the professor.
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## Subject's extra information

We recommend a good basis for the subjects of first year courses Calculus and Linear Algebra. The course requires continuous work throughout the semester to achieve their goals. It is also necessary critical thinking and capacity for abstraction. You can find the following materials in the Copisteria Campus Capped (Building Aulari) and the Virtual Campus <http://cv.udl.cat> Collection set of exercises, tests resolutions for previous years; Statements of Practice lab.

## Learning objectives

The course is based on obtaining constructive methods for the approximation of the solutions of real problems. Numerical methods are an essential tool in the field of applied science dealing approximate design methods, efficiently, solutions to problems previously formulated mathematically. In most cases, the mathematical problem stems from a practical problem in areas such as experimental engineering. The aim of the course is the study of algorithms and constructive methods that allow us to get the solution of a problem with arbitrary high precision in a finite number of steps. Since many calculations are required, the development of numerical methods has been in parallel with the computers that made their use feasible. The course will support technical subjects of the same course and other advanced courses.

## Competences

### Degree-specific competences

- Ability to resolve logical problems that can arise in engineering. Aptitude to apply knowledge about lineal algebra; geometry; differential geometry; differential and integral calculus; differential equations and partial derivatives; numerical methods, numerical algorithms; statistics and optimization.

#### Goals

- Manipulate mathematical expressions and calculate fluently
- Synthesize the statement of a problem with the objective of express it in mathematical form
- To use mathematical techniques to solve problems of particular relevance in engineering
- Reasoning and analyzing the numerical results obtained from a calculation

### Degree-transversal competences

- Ability for abstraction and critical, logical and logical reasoning.

#### Goals

- Synthesize the statement of a problem with the objective of putting it in mathematical form
- Ability to resolve problems and elaborate and defend arguments inside their field of study

#### Goals

- Use of mathematical techniques to solve problems
- Ability to analyse and synthesize.

#### Goals

- Reasoning and analyzing the numerical results obtained from a calculation

## Subject contents

### 1. Errors, Stability and Conditioning.

#### 1.1. Preliminaries.

1.1.1. Scientific computing and application areas.

1.1.2. Mathematical modeling, numerical simulation and algorithms.

#### 1.2. Errors.

1.2.1. Errors in the entry.

1.2.2. Floating Point Arithmetic: rounding errors.

1.2.3. Truncation or discretization errors.

1.2.4. Analysis of error propagation.

#### 1.3. Stability.

1.3.1. Numerical stability of algorithms.

1.3.2. Unstable numerical problems.

1.3.3. Well and ill-conditioned problems.

### 2. Polynomial Interpolation.

#### 2.1. Introduction.

2.1.1. Objectives of the interpolation.

2.1.2. Different types of interpolation.

#### 2.2. Polynomial Interpolation.

2.2.1. Existence and uniqueness of the interpolating polynomial.

2.2.2. Lagrange formula.

2.2.3. Divided difference scheme and Newton interpolation.

2.2.4. Polynomial interpolation error.

2.2.5. The problem of polynomial interpolation: Runge phenomenon.

### 3. Numerical Integration.

#### 3.1. Introduction.

3.1.1. Utility of numerical integration.

3.1.2. Interpolating integration.

## 3.2. Newton-Cotes formulas.

3.2.1. Special cases: trapezium rule, Simpson's rule, etc ...

3.2.2. Compose formulas of Newton-Cotes.

3.2.3. Error in simple and compose formulas.

## 3.3. Romberg method.

## 4. Ordinary Differential Equations.

### 4.1. Introduction.

4.1.1. Initial value Cauchy problem.

4.1.2. Theorem of existence and uniqueness of the Cauchy problem.

### 4.2. One step methods.

4.2.1. Euler's method.

4.2.2. Taylor methods.

4.2.3. Heun or modified Euler method.

4.2.4. Runge-Kutta methods.

## 5. Nonlinear Equations.

### 5.1. Introduction.

5.1.1. Non exactly solvable equations: transcendental case.

5.1.2. Bolzano's theorem: bisection algorithm.

### 5.2. Some iterative methods.

5.2.1. Newton-Raphson or the tangent method.

5.2.2. Secant method.

### 5.3. Nonlinear systems and Newton-Raphson method.

## 6. Approximation of Functions.

### 6.1. Introduction and Theoretical Foundations.

6.1.1 Objectives of the approach.

6.1.2. Types of approximation: polynomial, trigonometric, exponential.

6.1.3. Discrete and continuous approach.

6.1.4. Existence and uniqueness of the approximating function.

6.1.5. Euclidean norm: Least Squares Approximation.

6.1.6. Normal equations.

6.2. Overdetermined linear systems.

6.3. Data linearization

## Methodology

This course consists of theoretical lessons, classes and practical problems with the help of a computer. The lectures will present content, demonstrate some of the key results and also will emphasize learning objectives. Moreover, the problem classes are designed for problem solving and discussion of specific points that the student must first work independently. In practical classes will be resolved (through teamwork) engineering problems with the implementation of programs written in code Octave / Matlab.

## Development plan

Timing of the contents of the subject:

- Weeks 1 and 2 (Item 1. **Errors, stability and conditioning**)
- Weeks 3,4 and 5 (Item 2. **Polynomial interpolation**)
- Weeks 6, 7 and 8 (Item 3. **Numerical Integration**)
- Weeks 10, 11 and 12 (Item 4. **Ordinary differential equations**)
- Weeks 13 and 14 (Item 5. **Nonlinear Equations**)
- Week 15 and 16 (Item 6. **Approximation of functions**)

## Evaluation

### EVALUATION OF THE COURSE NUMERICAL METHODS

Degree in Mechanical Engineering

Degree in Automation and Industrial Electronic Engineering

The course consists of two parts, a theoretical and a practical one. The theoretical part has a weight of 80% and the remaining 20% is the practical one.

**Theoretical part:** It consists of two written tests (controls) based on the resolution of problems. Each exam has the same weight and the note of theory is obtained by calculating the average value of the notes of the two controls. If this theory mark is equal to or greater than 5 then, and only then, the mark of practices can be added.

**Practical part:** Each pair of students (classmates) must submit a report where an engineering problem is solved by calculations using the computer and programming in the language Octave / Matlab. Before being corrected the delivered report the student must pass a Minimum Test, on the contrary the practical part will be failed.

**Resit:** You can resit the exam only for the theoretical part and the Minimum Test. The resit will be made by controls. The student has the right (not the obligation) to resit each of the failed controls. In addition, the student can not resit to climb a control mark if it is already passed. Finally, remember that the obtained mark in the resit is

the final mark regardless of whether it is superior or not to the initially failed mark.

**Calculation of Final Mark:** All the following marks are given on the interval  $[0, 10]$ .

C1 = Mark of Control 1

C2 = Mark of Control 2

P = Practical mark

M = Final Mark

If  $(C1 + C2) / 2 \geq 5$  and  $P \geq 5$ , then  $M = 0.8 * (C1 + C2) / 2 + 0.2 * P$

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Timing and load percentage evaluation activities:

- **Week 9.** Exam Control 1 of the content developed in class from week 1 through 8. This activity contributes 40% of the total mark for the course.
- **Week 15.** Delivery of reports from the practical part. This activity contributes 20% of the total mark for the course.
- **Week 16.** Exam Control 2 of the class content developed from week 10 through 15. This activity contributes 40% of the total mark for the course.

## Bibliography

### Basic bibliography:

- Chavarriga, J., García, I.A. y Giné, J. *Manual de Métodos Numéricos*. Edicions de la Universitat de Lleida, Eines **35**, 1999.
- García, I.A. y Maza, S. *Métodos Numéricos: Problemas Resueltos y Prácticas*. Edicions de la Universitat de Lleida. Eines **62**, 2009.
- Aubanell, A., Benseny, A. y Delshams, D. *Eines Bàsiques de Càlcul Numèric*. Publicacions de la UAB.
- Kincaid, D. y Cheney, W. *Análisis numérico*. Ed. Addison-Wesley, Delaware, 1994.
- Grau, M. y Noguera, M. *Càlcul Numèric*. Ed. UPC, Barcelona, 1993.
- Burden, R.L y Douglas Faires, J. *Análisis Numérico*. 6a edición, International Thomson Editores, México, 1999.

### Recommended Bibliography:

- Dahlquist, G. and Björck, A. *Numerical methods*. Ed. Prentice-Hall, Englewood Cliffs, 1974.
- Isaacson, E. and Keller, H.B. *Analysis of Numerical Methods*. John Wiley, New York, 1966.
- Kress, R. *Numerical Analysis*. Ed. Springer-Verlag, New-York, 1998.

