



Universitat de Lleida

DEGREE CURRICULUM
LINEAR ALGEBRA

Coordination: GINE MESA, JAUME

Academic year 2023-24

Subject's general information

Subject name	LINEAR ALGEBRA			
Code	102100			
Semester	2nd Q(SEMESTER) CONTINUED EVALUATION			
Typology	Degree	Course	Character	Modality
	Bachelor's Degree in Automation and Industrial Electronic Engineering	1	COMMON/CORE	Attendance-based
	Bachelor's Degree in Energy and Sustainability Engineering	1	COMMON/CORE	Attendance-based
	Bachelor's Degree in Mechanical Engineering	1	COMMON/CORE	Attendance-based
	Common branch in industrial engineering programs - Lleida	1	COMMON/CORE	Attendance-based
	Double bachelor's degree: Degree in Mechanical Engineering and Degree in Energy and Sustainability Engineering	1	COMMON/CORE	Attendance-based
	Programa Acadèmic de Recorregut Successiu - Enginyeries Industrials	1	COMMON/CORE	Attendance-based
Course number of credits (ECTS)	6			
Type of activity, credits, and groups	Activity type	PRAULA	TEORIA	
	Number of credits	3	3	
	Number of groups	3	3	
Coordination	GINE MESA, JAUME			
Department	MATHEMATICS			
Teaching load distribution between lectures and independent student work	60 classroom hours and 90 hours of independent work.			

LINEAR ALGEBRA 2023-24

Important information on data processing	Consult this link for more information.
Language	Catalan
Distribution of credits	3 theoretical credits and 3 practical credits.

Teaching staff	E-mail addresses	Credits taught by teacher	Office and hour of attention
GINE MESA, JAUME	jaume.gine@udl.cat	18	By appointment

Subject's extra information

The course requires continuous work throughout the semester in order to achieve the objectives. It requires critical thinking and capacity for abstraction.

You can find collections of resolutions of exams of previous years (Technical Industrial Engineering) in the "Copisteria" of Capponet Campus (edifici de l'Aulari) and at the personal page <http://web.udl.es/usuarios/t4088454/ssd/>

It is recommended to visit the Virtual Campus of the Course frequently where advertises all the relevant information.

Learning objectives

Learning objectives:

- Discuss and solve linear systems of equations.
- Describe vector spaces and subspaces.
- Perform subspace operations: intersection, sum and direct sum..
- Describe linear applications from their kernel and their image. Perform changes of basis in a linear application.
- Determine whether or not an endomorphism diagonalizes.
- Apply the diagonalization of endomorphisms to determine powers of matrices, to solve linear recurrences, and differential systems of linear equations.
- Classify a bilinear form. Express the quadratic form associate as a sum of squares.
- Apply the scalar product of vectors to calculate distances and angles.
- Apply the orthonormalization Gram-Schmidt process in order to compute orthonormal basis.

Competences

EPS1. Capacity to solve problems and prepare and defence arguments inside the area of studies.

EPS5. Capacity of abstraction and of critical, logical and mathematical thinking.

EPS6. Capacity of analysis and synthesis.

GEM1. Capacity to solve mathematical problems arisen in the engineering field. Aptitude to apply knowledge on: linear algebra; geometry; differential geometry; differential and integral calculus; differential equations and in partial derivatives; numerical methods; algorithmic, numerical; statistics and optimisation.

Subject contents

1. Matrices, determinants and systems of linear equations.

- 1.1 Types of Matrices and operations with matrices.
- 1.2 Invertible matrices.
- 1.3 Determinant of a square matrix of order n . Cases $n=2$, $n=3$ and $n>3$.
- 1.4 Rank of a matrix.
- 1.5 Elementary transformations by rows. Gauss elimination method.
- 1.6 Expression of a system of linear equations by matrices.
- 1.7 Rouché-Frobenius theorem.
- 1.8 Cramer Systems

2. Vector spaces.

- 2.1 Definition, properties and examples.
- 2.2 Linear combination: linear dependence and linear independence.
- 2.3 Generating system.
- 2.4 Basis of a vector space: definition, dimensions, components.
- 2.5 Vector Subspace. Linear varieties.
 - 2.5.1 Description by using generators and equations.
 - 2.5.2 Gauss Transformations.
 - 2.5.3 Basis and dimension.
- 2.6 Changes of basis. Basis change matrix.
- 2.7 Operations on subspaces.
 - 2.7.1 Containment and equality.
 - 2.7.2 Intersection and sum.
 - 2.7.3 Grassman formula.
 - 2.7.4 Complementary and direct sum.

3. Linear transformations and endomorphism diagonalization.

- 3.1 Definition and properties of a linear transformation.
- 3.2 Determination by the image in a basis.
- 3.3 Kernel and image of a linear transformation.
- 3.4 Matrix representation of a linear transformation.
- 3.5 Changes of basis in a linear transformation. Similar matrices.

3.6 Operations with linear applications.

3.6.1 Sum of applications.

3.6.2 Product by a scalar.

3.6.3 Composition of applications.

4. Endomorphism diagonalization.

4.1 Diagonal matrix: eigenvalues and eigenvectors.

4.2 Characteristic polynomial of an endomorphism: definition and calculation.

4.3 Cayley-Hamilton theorem.

4.4 Algebraic multiplicity.

4.5 Subspace generated by an eigenvector: geometric multiplicity.

4.6 Characterization of diagonalizable endomorphisms.

4.7 Applications of diagonalization: Powers of matrices, Linear Recurrences and Systems of linear differential equations.

5. Bilinear forms and quadratic forms.

5.1 Properties of the ordinary scalar product in \mathbb{R}^n . Concept of bilinear form.

5.2 Matrix representation of a bilinear form in a base. Concept of orthogonality.

5.3 Symmetric bilinear form. Gauss method to find a orthogonal basis. Gram-Schmidt process of orthonormalization.

5.4 Definition of rank and signature. Effective methods of calculation of these invariants.

5.5 Scalar Products. Definition of the associated norms. Orthonormal basis. Applications to geometry.

5.6 Quadratic form associated to a bilinear form.

5.7 Applications: Expression of a quadratic form as a linear combination of squares. Classification of conics and quadrics.

Methodology

The activities will be divided into two parts that complement each other: lectures and resolution of problems classes.

Lectures: In the lectures introduce concepts and relevant theoretical results and illustrate them with examples and exercises.

Resolution of problems: these classes are devoted to difficult exercises in order to gradually consolidate the concepts and ideas developed in the lectures.

Both lectures and resolution of problems taught in small groups of students. Having smaller groups of students promotes dialogue and participation from them.

In addition, students will have the responsibility to strengthen their knowledge independently on the basis of the material provided or recommended by the teacher.

Development plan

Activities:

Lectures: The issues described in the section Contents will be held in the master classes throughout the different weeks of class.

From: Beginning of the semester until the end of the semester. Classroom hours 28 h Learning hours 42 h

Resolution of problems: The issues described in section contents is developed in practices in the classroom during the teaching weeks.

From: Beginning of the semester until the end of the semester. Classroom hours 28 h learning hours 42 h

Practices in the classroom: Exercises in the classroom that are delivered to the teacher.

At the end of each semester. 1 h contact hours

Tutorial: Revisions examinations and office hours serve to resolve doubts and clarify concepts.

From: Beginning of the semester until the end of the semester.

Evaluation: The evaluation described in paragraph Evaluation, written tests will be developed through classroom (exams) and delivery of exercises.

From: Beginning of the semester until the end of the semester. Contact hours 6h hours.

Week	Methodology	Syllabus	Presential Hours	Learning hours
1-8	Lectures	Chapter 1,2,3	14	21
1-8	Resolution of problems	Chapter 1,2,3	14	21
8	Practices in the classroom	Chapter 1,2,3	1	
9	Evaluation. Written test	Chapter 1,2,3	3	
10-15	Lectures	Chapter 4,5	14	21
10-15	Resolution of problems	Chapter 4,5	14	21
15	Practices in the classroom	Chapter 4.5	1	
16	Evaluation. Written test	Tema 4,5	3	

Evaluation

Activity: Written test Dates: Week 9 percentage: 45% Character: Compulsory Realization: Single

Criteria: Written exam - PA1 To consider the note of exercise delivery must be at least 4 (10) of PA1 and PA2.
Note: In written tests can take conventional scientific calculator. But not authorized the use of these calculators, or other device, which carry information stored or can transmit it.

Exercise delivery on the virtual campus Dates: Week 8 Percentatge 5% Character: Compulsory Realization: Single

Criteria: To consider the note of exercise delivery must be at least 4 (10) of PA1 and PA2. There will be random validation of the exercise and failure to pass the validation will involve suspending the subject by copy.

Activity: Written test Dates: Week 16 Percentage: 45% Character: Compulsory Realization: Single

Criteria: Written exam - PA2 To consider the note of exercise delivery must be at least 4 (10) of PA1 and PA2.

Note: In written tests can take conventional scientific calculator. But not authorized the use of these calculators, or other device, which carry information stored or can transmit it.

Exercise delivery on the virtual campus Dates: Week 15 Percentatge 5% Character: Compulsory Realization: Single

Criteria: To consider the note of exercise delivery must be at least 4 (10) of PA1 and PA2. There will be random validation of the exercise and failure to pass the validation will involve suspending the subject by copy.

Activity: Problems Dates: Week 15 Percentatge 10% Character: Volunteer Realization: Single

Criteria: To consider the note of Problems should be at least 4 (10) of PA1 and PA2.

Activity: Recovery exam Dates: Week set by the EPS. Percentage 80% Recovering the two written tests. **Criteria:** Written exam - R-

In order to take into account the grade of delivery of the exercise you must have at least a 4 (out of 10) of R.

Alternative Evaluation: Dates: Week 16 and week set by the EPS. This assessment consists of written test PA1+PA2 in week 16. Percentage 100% and if recovery is necessary written test R Percentage 100%.

Bibliography

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REVIEWING BIBLIOGRAPHY

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