



Universitat de Lleida

DEGREE CURRICULUM  
**LINEAR ALGEBRA**

Coordination: GINE MESA, JAUME

Academic year 2016-17

## Subject's general information

<b>Subject name</b>	LINEAR ALGEBRA			
<b>Code</b>	102100			
<b>Semester</b>	2nd Q(SEMESTER) CONTINUED EVALUATION			
<b>Typology</b>	<b>Degree</b>	<b>Course</b>	<b>Typology</b>	<b>Modality</b>
	Bachelor's Degree in Automation and Industrial Electronic Engineering	1	COMMON	Attendance-based
	Bachelor's Degree in Mechanical Engineering	1	COMMON	Attendance-based
<b>ECTS credits</b>	6			
<b>Groups</b>	2GG,4GM			
<b>Theoretical credits</b>	3			
<b>Practical credits</b>	3			
<b>Coordination</b>	GINE MESA, JAUME			
<b>Department</b>	MATEMATICA			
<b>Important information on data processing</b>	Consult <a href="#">this link</a> for more information.			
<b>Language</b>	Catalan			
<b>Office and hour of attention</b>	by appointment			

Professor/a (s/es)	Adreça electrònica professor/a (s/es)	Crèdits	Horari de tutoria/lloc
GINE MESA, JAUME	gine@matematica.udl.cat	18	By appointment

## Subject's extra information

The course requires continuous work throughout the semester in order to achieve the objectives. It requires critical thinking and capacity for abstraction.

You can find collections of resolutions of exams of previous years (Technical Industrial Engineering) in the "Copisteria" of Cappon Campus (edifici de l'Aulari) and at the personal page <http://web.udl.es/usuarios/t4088454/ssd/>

It is recommended to visit the Virtual Campus of the Course frequently where advertises all the relevant information.

## Learning objectives

Learning objectives:

- Discuss and solve linear systems of equations.
- Describe vector spaces and subspaces.
- Perform subspace operations: intersection, sum and direct sum..
- Describe linear applications from their kernel and their image. Perform changes of basis in a linear application.
- Determine whether or not an endomorphism diagonalizes.
- Apply the diagonalization of endomorphisms to determine powers of matrices, to solve linear recurrences, and differential systems of linear equations.
- Classify a bilinear form. Express the quadratic form associate as a sum of squares.
- Apply the scalar product of vectors to calculate distances and angles.
- Apply the orthonormalization Gram-Schmidt process in order to compute orthonormal basis.

## Competences

**EPS1. Capacity to solve problems and prepare and defence arguments inside the area of studies.**

**EPS5. Capacity of abstraction and of critical, logical and mathematical thinking.**

**EPS6. Capacity of analysis and synthesis.**

**GEM1. Capacity to solve mathematical problems arisen in the engineering field. Aptitude to apply knowledge on: linear algebra; geometry; differential geometry; differential and integral calculus; differential equations and in partial derivatives; numerical methods; algorithmic, numerical; statistics and optimisation.**

## Subject contents

## **1. Matrices, determinants and systems of linear equations.**

- 1.1 Types of Matrices and operations with matrices.
- 1.2 Invertible matrices.
- 1.3 Determinant of a square matrix of order  $n$ . Cases  $n=2$ ,  $n=3$  and  $n>3$ .
- 1.4 Rank of a matrix.
- 1.5 Elementary transformations by rows. Gauss elimination method.
- 1.6 Expression of a system of linear equations by matrices.
- 1.7 Rouché-Frobenius theorem.
- 1.8 Cramer Systems

## **2. Vector spaces.**

- 2.1 Definition, properties and examples.
- 2.2 Linear combination: linear dependence and linear independence.
- 2.3 Generating system.
- 2.4 Basis of a vector space: definition, dimensions, components.
- 2.5 Vector Subspace. Linear varieties.
  - 2.5.1 Description by using generators and equations.
  - 2.5.2 Gauss Transformations.
  - 2.5.3 Basis and dimension.
- 2.6 Changes of basis. Basis change matrix.
- 2.7 Operations on subspaces.
  - 2.7.1 Containment and equality.
  - 2.7.2 Intersection and sum.
  - 2.7.3 Grassman formula.
  - 2.7.4 Complementary and direct sum.

## **3. Linear transformations and endomorphism diagonalization.**

- 3.1 Definition and properties of a linear transformation.
- 3.2 Determination by the image in a basis.
- 3.3 Kernel and image of a linear transformation.
- 3.4 Matrix representation of a linear transformation.
- 3.5 Changes of basis in a linear transformation. Similar matrices.
- 3.6 Operations with linear applications.
  - 3.6.1 Sum of applications.

3.6.2 Product by a scalar.

3.6.3 Composition of applications.

## 4. Endomorphism diagonalization.

4.1 Diagonal matrix: eigenvalues and eigenvectors.

4.2 Characteristic polynomial of an endomorphism: definition and calculation.

4.3 Cayley-Hamilton theorem.

4.4 Algebraic multiplicity.

4.5 Subspace generated by an eigenvector: geometric multiplicity.

4.6 Characterization of diagonalizable endomorphisms.

4.7 Applications of diagonalization: Powers of matrices, Linear Recurrences and Systems of linear differential equations.

## 5. Bilinear forms and quadratic forms.

5.1 Properties of the ordinary scalar product in  $\mathbb{R}^n$ . Concept of bilinear form.

5.2 Matrix representation of a bilinear form in a base. Concept of orthogonality.

5.3 Symmetric bilinear form. Gauss method to find a orthogonal basis. Gram-Schmidt process of orthonormalization.

5.4 Definition of rank and signature. Effective methods of calculation of these invariants.

5.5 Scalar Products. Definition of the associated norms. Orthonormal basis. Applications to geometry.

5.6 Quadratic form associated to a bilinear form.

5.7 Applications: Expression of a quadratic form as a linear combination of squares. Classification of conics and quadrics.

## Methodology

The activities will be divided into two parts that complement each other: lectures and resolution of problems classes.

**Lectures:** In the lectures introduce concepts and relevant theoretical results and illustrate them with examples and exercises.

**Resolution of problems:** these classes are devoted to difficult exercises in order to gradually consolidate the concepts and ideas developed in the lectures.

Both lectures and resolution of problems taught in small groups of students. Having smaller groups of students promotes dialogue and participation from them.

In addition, students will have the responsibility to strengthen their knowledge independently on the basis of the material provided or recommended by the teacher.

## Development plan

## Activities:

**Lectures:** The issues described in the section Contents will be held in the master classes throughout the different weeks of class.

From: Beginning of the semester until the end of the semester. Classroom hours 28 h Learning hours 42 h

**Resolution of problems:** The issues described in section contents is developed in practices in the classroom during the teaching weeks.

From: Beginning of the semester until the end of the semester. Classroom hours 28 h learning hours 42 h

**Practices in the classroom:** Exercises in the classroom that are delivered to the teacher.

At the end of each semester. 1 h contact hours

**Tutorial:** Revisions examinations and office hours serve to resolve doubts and clarify concepts.

From: Beginning of the semester until the end of the semester.

**Evaluation:** The evaluation described in paragraph Evaluation, written tests will be developed through classroom (exams) and delivery of exercises.

From: Beginning of the semester until the end of the semester. Contact hours 6h hours.

Week	Methodology	Syllabus	Presential Hours	Learning hours
1-8	Lectures	Chapter 1,2,3	14	21
1-8	Resolution of problems	Chapter 1,2,3	14	21
8	Practices in the classroom	Chapter 1,2,3	1	
9	Evaluation. Written test	Chapter 1,2,3	3	
10-15	Lectures	Chapter 4,5	14	21
10-15	Resolution of problems	Chapter 4,5	14	21
15	Practices in the classroom	Chapter 4.5	1	
16	Evaluation. Written test	Tema 4,5	3	

## Evaluation

**Activity: Written test** Dates: Week 8 percentage: 40% Character: Compulsory Realization: Single

Criteria: Written exam - PA1 To consider the note of attendance and participation must be at least 4 (10) of PA1 and PA2.

Note: In written tests can take conventional scientific calculator. But not authorized the use of these calculators, or other device, which carry information stored or can transmit it.

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**Activity: Participation and Attendance** Dates: 1/8 Weeks Percentatge 10% Character: Compulsory Realization: Single

Criteria: To consider the note of attendance and participation must be at least 4 (10) of PA1 and PA2.

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**Activity: Written test:** Dates: Week 17/18 Percentage: 40% Character: Compulsory Realization: Single

Criteria: Written exam - PA2 To consider the note of attendance and participation must be at least 4 (10) of PA1 and PA2.

Note: In written tests can take conventional scientific calculator. But not authorized the use of these calculators, or other device, which carry information stored or can transmit it.

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**Activity: Participation and Attendance** Dates: 9/16 Weeks Percentatge 10% Character: Compulsory Realization: Single

Criteria: To consider the note of attendance and participation must be at least 4 (10) of PA1 and PA2.

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**Activity: Problems** Dates: Week 16 Percentatge 10% Character: Volunteer Realization: Single

Criteria: To consider the note of Problems should be at least 4 (10) of PA1 and PA2.

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## Bibliography

### BASIC BIBLIOGRAPHY

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I.A.García i J. Giné, Problemas resueltos de Àlgebra Lineal. Col.lecció Eines, no.45. Ed. de la UdL. Lleida. 2003.

### REVIEWING BIBLIOGRAPHY

J. Sorolla Bardají, Introducció a la Matemàtica, ISBN: 978-84-616-4854-2

### RECOMMENDED BIBLIOGRAPHY

M.Alsina, J.M. Miret i A. Rio, Càlcul i Àlgebra. Resums i Problemes (ambsolucions), 1991.

J.Arvesú, F. Marcellán i J. Sánchez, Problemas resueltos de álgebra lineal., Madrid, International Thomson, 2005.

C.Alsina i E. Trillas, Lecciones de Àlgebra y Geometría. Editorial Gustavo Gili, S.A., Barcelona, 1984.

M.Castellet i I. Llerena, Àlgebra lineal i geometria., Publicacions de la UAB, Barcelona, 1988.

P.M.Cohn, Elements of linear algebra, Chapman and Hall, London, 1994.

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M.T.García González, A. Ruiz Oliarria, M.M. Saiz Jarabo, Álgebra. Teoría y ejercicios. Ed. Paraninfo, Madrid, 1993.

J.Gimbert i J.M. Miret, Problemes d'Àlgebra per a Ciències de la Computació. Col.lecció Eines, no. 20. Ed. de la UdL. Lleida. 1997.

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