



Universitat de Lleida

## DEGREE CURRICULUM

# DISCRETE MATHEMATICS

Coordination: LOPEZ LORENZO, IGNACIO

Academic year 2021-22

## Subject's general information

|  |   |        |           |                  |
|--|---|--------|-----------|------------------|
| Subject name   | DISCRETE MATHEMATICS  |        |           |                  |
| Code   | 102007  |        |           |                  |
| Semester   | 1st Q(SEMESTER) CONTINUED EVALUATION  |        |           |                  |
| Typology   | Degree  | Course | Character | Modality         |
|  | Bachelor's Degree in Computer Engineering   | 2      | COMMON    | Attendance based |
|  | Double bachelor's degree: Degree in Computer Engineering and Degree in Business Administration and Management | 2      | COMMON    | Attendance-based |
| Course number of credits (ECTS)  | 6   |        |           |                  |
| Type of activity, credits, and groups                                    | Activity type   | PRAULA | TEORIA    |                  |
|  | Number of credits   | 3      | 3         |                  |
|  | Number of groups  | 2      | 1         |                  |
| Coordination   | LOPEZ LORENZO, IGNACIO  |        |           |                  |
| Department   | MATHEMATICS   |        |           |                  |
| Teaching load distribution between lectures and independent student work | 6 ECTS = 25x6 = 150 total hours.<br>40% --> 60 classroom hours.<br>60% --> 90 student study hours.            |        |           |                  |
| Important information on data processing                                 | Consult <a href="#">this link</a> for more information.   |        |           |                  |
| Language   | Basically in Catalan. Spanish language will be used if there are students with zero knowledge of Catalan.     |        |           |                  |
| Distribution of credits  | Nacho López 10,5<br>Pol Llagostera 1,5  |        |           |                  |

| Teaching staff                 | E-mail addresses              | Credits taught by teacher | Office and hour of attention |
|--------------------------------|-------------------------------|---------------------------|------------------------------|
| CERESUELA TORRES, JESUS MIGUEL | jesusmiguel.ceresuela@udl.cat | 2                         |                              |
| LOPEZ LORENZO, IGNACIO         | nacho.lopez@udl.cat           | 7                         |                              |

## Subject's extra information

Discrete mathematics is the study of mathematical structures that are fundamentally discrete rather than continuous. Mathematically speaking, the term discrete means that the elements are somehow split away between them. For instance, integers and discrete algebra are part of discrete mathematics (and they have been introduced in Algebra subject). Combinatorics and graph theory are also discrete mathematics and they are part of this subject. Besides, there are many others topics in discrete mathematics, like code theory, cryptography, etc. but they appear in other subjects of the degree. The main reason to put combinatorics and graph theory in this subject is because they have a lot of applications in computer science, in fact, data in the computers is stored meanwhile in a discrete way (through binary sequences of 0's and 1's). This course is a first approach to Graph Theory and Combinatorics.

To follow this subject properly some previous knowledge on basic Algebraic group theory is recommended.

## Learning objectives

- Model real problems through graphs.
- To know the basic parameters of a graph and different representations.
- To determine whenever two small graphs are isomorphic
- Distinguish between DFS and BFS strategies.
- To determine whenever a graph is connected.
- To know some connectivity parameters.
- To compute related distance graph parameters.
- To know algorithms in order to compute distances in weighted graphs and non-weighted graphs.
- To compute walks in graphs and related concepts.
- To prove if a given graph is an Eulerian graph, and if it is so, to find an eulerian circuit.
- To study the hamiltonicity of a graph.
- To identify Trees and their elementary properties
- To recognize in which situations an optimum coloration in a graph is needed.
- To evaluate the efficiency of some basic algorithms related to graphs.
- To know elementary combinatorics.
- To model some counting problems and solving them through combinatorics.
- To know permutations, combinations and variations.
- To apply combinatoric formulas.
- To know the Inclusion–exclusion principle
- To know recurrence relations.
- To solve recurrence linear equations of order two with constant coefficients.

## Competences

Strategic Competitions of the UdL according to the “Plan Director de la Docència” approved by the Government Council of UdL on July 10th, 2007.

- CT5. Acquire knowledge in scientific thinking.

Cross-disciplinary competences approved by the Plenary Commission of the Degrees of Industrial Engineering, Computer Engineering and Building Engineering, gathered on June 16th, 2008.

- EPS1. Capacity to solve problems and prepare and defence arguments inside the area of studies.
- EPS5. Capacity of abstraction and of critical, logical and mathematical thinking.

Specific competences that the students have to acquire in the degree in Computer Engineering set in the Royal decree 1393/2007, of October 29th

- GII-FB1. Capacity to solve mathematical problems arisen in the engineering field. Aptitude to apply knowledge on: linear algebra; differential and integral calculus; numerical methods; algorithmic, numerical; statistics and optimisation.
- GII-FB3. Capacity to understand and master the basic concepts of discrete mathematics, logical, algorithmic and computational complexity, and its application to solve engineering problems.

## Subject contents

### I. INTRODUCTION TO COMBINATORICS.

#### 1. Elementary combinatoric topics.

1.0 Introduccion.

1.1 Basic principles in combinatorics.

1.2 Permutations

1.3 Combinations.

1.4 Binomial coefficients. Multinomial coefficients.

1.5 Special counting methods: Inclusion and Exclusion.

#### 2. Recurrence relations.

2.0 Introduction.

2.1 Basic terminology on recurrence relations.

2.2 Methods for solving recurrence relations.

2.3 Solving linear recurrence relations of order two with constant coefficients.

### II. BASICS ON GRAPH THEORY

#### 1. Graphs and related objects.

1.0 Graph as mathematical models.

1.1 Definition of a graph.

1.2 Degree of a vertex. Hand shaking lemma.

1.3 Graph representations.

1.4 Graph isomorphism.

1.5 Main graph examples.

1.6 Operations with graphs.

1.7 Directed graphs

1.8 Introduction to software SAGE

## 2. Connectivity and distances

2.1 Walks in graphs.

2.2 Connected graphs. DFS algorithm.

2.3 Graph connectivity.

2.4 Distances in graphs. BFS algorithm.

2.5 Trees.

## 3. Eulerian graphs and hamiltonian graphs

3.1 Eulerian graphs: characterization.

3.2 Hierholzer algorithm and Fleury algorithm.

3.3 Necessary and sufficient condition on hamiltonian graphs.

## 4. Approaching other topics in graph theory

4.1 Planarity.

4.2 Coloring.

## Methodology

### Big groups: Classes de Teoria (3 crèdits)

- Part teòrica: classes suportades amb apunts i material disponible al campus virtual.
- Part d'aplicació pràctica: es treballa sempre amb exemples i exercicis. Es disposa d'una **col·lecció de problemes**, dels quals es van proporcionant solucions al llarg del quadrimestre.

### Small groups: Classes de Problemes (3 crèdits)

- Classes dirigides i seguiment de la resolució de problemes

### Student non-classroom work:

- Es recomana que l'alumne resolgui per compte propi els problemes de la col·lecció de problemes que no es resolguin a classe, amb la finalitat de completar el coneixement teòric i pràctic.

## Development plan

| Dates<br>(Week number) | Description                    | Activity in big group                   | Activity in small group | Student work                   |
|------------------------|--------------------------------|---|-------------------------|--------------------------------|
| 1-8                    | Contents of Section I          | Master classes and problems resolution. | Problems resolution.    | Study and problems resolution. |
| 5                      | Solve a problem.               | Evaluation exercise.                    | Evaluation exercise.    | Study and problems resolution. |
| 9                      | Exam of contents of section I  | Evaluation part.                        | Evaluation part.        | Study and problems resolution. |
| 10-16                  | Contents of Section II         | Master classes and problems resolution. | Problems resolution.    | Study and problems resolution. |
| 12                     | Programming in SAGE            | Evaluation exercise.                    | Evaluation exercise.    | Programming in SAGE            |
| 17-18                  | Exam of contents of section II | Evaluation part.                        | Evaluation part.        | Study and problems resolution. |
| 20                     | Exam.                          | Recuperation                            | Recuperation            | Study and problems resolution. |

## Evaluation

The evaluation of discrete mathematics is as follows:

- First partial exam (week number 9), weighted on 4 points, where contents of section 1.
- Second partial exam (week number 17-18), weighted on 4 points, where the rest of the contents will be evaluated (also the basic results of the first section).
- Resolution of two problems either after and before the first partial exam, weighted on 1 point each.
- There is a recuperation exam, weighted on 8 points, for those students whose qualification is lower than 5 with the sum of qualifications in the points given above.

| Acr. | Evaluation activities | Weight | Recuperation |
|------|-----------------------|--------|--------------|
| P1   | First partial exam    | 40%    | YES          |
| P2   | Second partial exam   | 40%    | YES          |
| PRA  | Problems' resolution  | 10%    | NO           |
| PRB  | Problems' resolution  | 10%    | NO           |

$$\text{FinalQualification} = 0,4*P1 + 0,4*P2 + 0,1*PRA + 0,1*PRB$$

## Bibliography

### Available In Campus Virtual

Related to Combinatorics

- Gimbert, J., Moreno R., Valls M., Notes sobre Combinatòria, Quadern EUP núm. 36, 2002. (en catalán).

Related to Graph Theory

- Gimbert, J., Moreno, R., Ribó, J.M., Valls, M., Apropament a la Teoria de Grafs i als seus Algorismes, Edicions de la UdL, 1998. (In catalán).
- Bijedi, N., Gimbert J., Miret J.M., Valls M., Elements of Discrete Mathematical Structures for Computer Science, Univerzitetska knjiga Mostar and Edicions de la UdL, 2007. (In english)

### Basic Bibliography

THEORY BOOKS (with some problems to solve)

- Anderson, I., Introducció a la Combinatoria. Vicens Vives, 1993.
- Brunat, J.M., Combinatòria i Teoria de Grafs. Edicions UPC, 1996.
- Biggs, N., Matemàtica Discreta. Vicens Vives, 1993.

PRACTICE BOOKS (with solved problems)

- Bijedi, N., Gimbert J., Miret J.M., Valls M., Elements of Discrete Mathematical Structures for Computer Science, Univerzitetska knjiga Mostar and Edicions de la UdL, 2007.
- García, F., Hernández, G., Nevot, A., Problemas resueltos de Matemática Discreta. Thomson, 2003.
- Trias, J., Matemàtica Discreta. Problemes resolts. Edicions UPC, 2001.