

## DEGREE CURRICULUM LINEAR ALGEBRA

Coordination: GRAU MONTAÑA, MARIA TERESA

Academic year 2023-24

## Subject's general information

| Subject name | LINEAR ALGEBRA |  |  |  |  |
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| Code | 101401 |  |  |  |  |
| Semester | 2nd Q(SEMESTER) CONTINUED EVALUATION |  |  |  |  |
| Typology | Degree |  | Course | Character | Modality |
|  | Bachelor's Degree in Architectural Technology and Building Construction |  | 1 | COMMON/CORE | Attendancebased |
| Course number of credits (ECTS) | 6 |  |  |  |  |
| Type of activity, credits, and groups | Activity type | PRAULA |  | TEORIA |  |
|  | Number of credits | 3 |  | 3 |  |
|  | Number of groups | 1 |  | 1 |  |
| Coordination | GRAU MONTAÑA, MARIA TERESA |  |  |  |  |
| Department | MATHEMATICS |  |  |  |  |
| Teaching load distribution between lectures and independent student work | 40\% at the classroom, $60 \%$ autonomous work See "Development plan". |  |  |  |  |
| Important information on data processing | Consult this link for more information. |  |  |  |  |
| Language | Catalan |  |  |  |  |


| Teaching staff | E-mail addresses | Credits <br> taught by <br> teacher | Office and hour of attention |
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| GRAU MONTAÑA, MARIA <br> TERESA | maite.grau@udl.cat | 6 |  |

## Subject's extra information

To follow this subject properly the previous knowledge taught in the subject area of Mathematics of the "Bachillerato Tecnológico" (see https://xtec.gencat.cat/ca/curriculum/batxillerat/) is needed. In particular , the sections on arithmetic and algebra and geometry.

## Suggestions

Subject that requires continuous work throughout the semester in order to achieve its goals. It requires critical thinking and capacity for abstraction. You can find collections of the materials at the Cappont Campus (Building Aulari) and the Virtual Campus: https://cv.udl.cat

- Collection set of exercises with the numerical solutions.
- Resolutions of exams in previous years.

It is recommended to frequently visit the site at the Virtual Campus since all the information is announced there.

## The course as part of the academic plan

This course is in the 2nd semester of 1st year of teaching. It belongs to the module of "Basic Training", specifically in the field "Scientific Basis".

## Learning objectives

- Perform elementary Gauss transformations in order to determine the rank of a matrix.
- Apply the Rouché-Frobenius Theorem and solve systems of linear equations.
- Describe vector space and vector subspace.
- Make vector calculations in different bases. In particular, calculating the components of a vector on different bases.
- Determine a linear map from its image on a base.
- Calculate the kernel and image of a linear map.
- Determine the eigenvalues and eigenvectors associated with an endomorphism and interpret them geometrically.
- Determine, classify and construct matrices of orthogonal transformations in the plane and in space.
- Apply vector calculus in the description of geometric objects.
- Classify conics and quadrics from its equation.


## Competences

## Cross-disciplinary competences of the degree:

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EPS1. Capacity to solve problems and prepare and defence arguments inside the area of studies.
EPS2. Capacity to gather and interpret relevant data, within the area of study, to judge and think about relevant subjects of social, scientific and ethical nature.

EPS5. Capacity of abstraction and of critical, logical and mathematical thinking.
EPS6. Capacity of analysis and synthesis.

## Specific competences of the degree:

GEE1. Aptitude to apply the knowledge related with numerical and infinitesimal calculus, linear algebra, analytical and differential geometry, techniques and probabilistic methods and statistical analysis.

GEE2. Applied knowledge of the principles of general mechanics, the statics of structural systems, the geometry of masses, the principles and methods of analysis of the elastic behaviour of the solid.

## Subject contents

1. Matrices, determinants and systems of linear equations.
1.1 Types of matrices and matrix operations.
1.2 Inversible matrices.
1.3 Determinant of a square matrix of order $n$. Cases $n=2, n=3$ and $n>3$.
1.4 Rank of a matrix.
1.5 Elementary transformations by rows. Gauss elimination method.
1.6 Expression of a system of linear equations by matrices.
1.7 Rouché-Frobenius theorem.
1.8 Cramer Systems

## 2. Vector spaces.

2.1 Definition, properties and examples.
2.2 Linear combination: linear dependence and linear independence.
2.3 Generating system.
2.4 Basis of a vector space: definition, dimensions, components.
2.5 Vector Subspace. Linear varieties.
2.5.1 Description by using generators and equations.

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2.5.2 Gauss Transformations.
2.5.3 Basis and dimension.
2.6 Basis change: matrix.
2.7 Operations on subspaces.
2.7.1 Containment and equality.
2.7.2 Intersection and sum.
2.7.3 Grassman formula.
2.7.4 Complementary and direct sum.

## 3. Linear transformations and endomorphism diagonalization.

3.1 Definition and properties of a linear transformation.
3.2 Determination by the image in a basis.
3.3 Kernel and image of a linear transformation.
3.4 Matrix representation of a linear transformation.
3.5 Changes of basis. Similar matrices.
3.6 Operations with linear applications.
3.6.1 Sum of applications.
3.6.2 Product by a scalar.
3.6.3 Composition of applications.
3.7 Diagonal matrix: eigenvalues and eigenvectors.
3.8 Characteristic polynomial of an endomorphism: definition and calculation.
3.9 Cayley-Hamilton theorem.
3.10 Algebraic multiplicity.
3.11 Subspace generated by an eigenvector: geometric multiplicity.
3.12 Characterization of diagonalizable endomorphisms.

## 4. Euclidean Geometry

4.1 Scalar product.
4.2 Norm induced by a scalar product.
4.3 Distances and angles.

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4.4 Concepts of orthogonal and orthonormal.
4.5 Gram-Schmidt process of orthonormalization.
4.6 Gram's determinant. Volume of cuboid.
4.7 Vector Product.
4.8 Orthogonal transformations: in the plane and in space.
4.9 Isometries or rigid motions.

## 5. Classification of conics and quadrics.

5.1 symmetric endomorphisms and scalar product.
5.2 symmetric bilinear form.
5.3 Range and signature: definition and calculation of these invariants.
5.4 Quadratic forms.
5.5 Quadratic varieties: canonical equation.
5.6 Classification of Euclidean conics and quadrics.
5.7 Other examples of curves. Trajectories. Parametrized curves.
5.8 Homothetic transformations.
5.9 Similarities.

## Methodology

Lectures: Classes to explain the theory and problem solving on the blackboard.

Resolution of problems: students solve problems individually during these sessions under the supervision of teachers of the subject.

Exam: two written tests are held during the semester. There is also a final retrieval test.

Follow-up tests: two follow-up written tests are carried out during the semester.

## Development plan

| Week | Methodology | Contents | Hours at classroom | Hours of <br> autonomous work |
| :--- | :--- | :--- | :--- | :--- |
| Weeks $1-3$ | Lectures | Chapter 1 | 6 | 9 |
| Weeks $1-3$ | Resolution of <br> problems | Chapter 1 | 6 | 9 |
| Weeks 4-6 | Lectures | Chapter 2 | 6 | 9 |


| Weeks 4-6 | Resolution of problems | Chapter 2 | 5 | 9 |
| :---: | :---: | :---: | :---: | :---: |
| Week 6 | Follow-up test | Chapters 1 and 2 | 1 | 0 |
| Weeks 7 and 8 | Lectures | Chapter 3 (sections $3.1-3.6)$ | 4 | 6 |
| Weeks 7 and 8 | Resolution of problems | Chapter 3 (sections $3.1-3.6)$ | 4 | 9 |
| Week 9 | $1^{\text {a }}$ test of evaluation (exam) | Contents until the moment | 1.5 |  |
| Weeks 10-12 | Lectures | Chapter 3 (sections $3.7-3.12)$ | 6 | 9 |
| Weeks 10-12 | Resolution of problems | Chapter 3 (sections $3.7-3.12)$ | 6 | 9 |
| Weeks 13 and 14 | Lectures | Chapter 4 | 4 | 6 |
| Weeks 13 and 14 | Resolution of problems | Chapter 4 | 3 | 6 |
| Week 13 | Follow-up test | Chapters 3 and 4 | 1 | 0 |
| Week 15 | Lectures | Chapter 5 | 2 | 3 |
| Week 15 | Resolution of problems | Chapter 5 | 2 | 6 |
| Weeks 16 and 17 | $2^{\text {a }}$ test of evaluation (exam) | Contents since the first test and until the moment | 1.5 |  |
| Week 20 | Retrieval test | All the contents of the subject | 2 |  |

## Evaluation

All assessment activities are face-to-face, individual and compulsory.

## Timing and percentage load of assessment activities:

PS1. Week 6. Follow-up written test. Delivery of an exercise proposed by the teacher and solved in class by the student. This activity
contributes $10 \%$ of the total mark of the subject
PA1. Week 9, following the school's academic calendar. written test Examination of the contents developed in class in the weeks from 1 to 9 . This activity contributes with $40 \%$ of the total mark of the subject.

PS2. Week 13. Follow-up written test. Delivery of an exercise proposed by the teacher and solved in class by the student. This activity
contributes $10 \%$ of the total mark of the subject.
PA2. Week 16, following the school's academic calendar. written test Examination of the contents developed in class in the weeks from 10 to 15 . This activity contributes with $40 \%$ of the total mark of the subject.

The final mark is configured with the marks of each of the four mentioned tests with their corresponding percentage. The subject will remain

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passed if the grade is a value equal to or higher than 5.
Resit exam. End of year, following the school's academic calendar. Examination of all the contents of the subject (all the topics). This activity counts for $80 \%$ of the total grade of the subject. All students can take this test to recover/improve their grade.

Alternative assessment. The student who is granted the option of alternative assessment must take the two partial exams (PA1 and PA2) on the dates set in the school's exam calendar. Each of these tests will count for $50 \%$ of the final grade. This student can also take the resit exam on the date set by the school, which will count for $100 \%$ of the final grade.

## Bibliography

## BASIC BIBLIOGRAPHY

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