

# DEGREE CURRICULUM LINEAR ALGEBRA

Coordination: PUJOLAS BOIX, JORDI

Academic year 2021-22

## Subject's general information

Subject name	LINEAR ALGEBRA					
Code	101401					
Semester	2nd Q(SEMESTER) CONTINUED EVALUATION					
Typology	Degree Cou			Character	Modality	
	Bachelor's Degree in Architectural Technology and Building Construction		1	COMMON	Attendance- based	
Course number of credits (ECTS)	6					
Type of activity, credits, and groups	Activity type	PRAULA	PRAULA		TEORIA	
	Number of credits	3		3		
	Number of groups	1			1	
Coordination	PUJOLAS BOIX, JORDI					
Department	MATHEMATICS					
Teaching load distribution between lectures and independent student work	40% at the classroom, 60% autonomous work See "Development plan".					
Important information on data processing	Consult this link for more information.					
Language	Catalan					

Teaching staff	E-mail addresses	Credits taught by teacher	Office and hour of attention
PUJOLAS BOIX, JORDI	jordi.pujolas@udl.cat	6	

#### Subject's extra information

To follow this subject properly the previous knowledge taught in the subject area of Mathematics of the "Bachillerato Tecnológico" (see <a href="http://xtec.gencat.cat/ca/curriculum/batxillerat/curriculum/">http://xtec.gencat.cat/ca/curriculum/</a> batxillerat/curriculum/) is needed. In particular, the sections on arithmetic and algebra and geometry.

#### Suggestions

Subject that requires continuous work throughout the semester in order to achieve its goals. It requires critical thinking and capacity for abstraction. You can find collections of the materials at the Cappont Campus (Building Aulari) and the Virtual Campus: http://cv.udl.cat

- Collection set of exercises with the numerical solutions.
- Resolutions of exams in previous years.

It is recommended to frequently visit the site at the Virtual Campus since all the information is announced there.

#### The course as part of the academic plan

This course is in the 2nd semester of 1st year of teaching. It belongs to the module of "Basic Training", specifically in the field "Scientific Basis".

## Learning objectives

- Perform elementary Gauss transformations in order to determine the rank of a matrix.
- Apply the Rouché-Frobenius Theorem and solve systems of linear equations.
- Describe vector space and vector subspace.
- Make vector calculations in different bases. In particular, calculating the components of a vector on different bases.
- Determine a linear map from its image on a base.
- Calculate the kernel and image of a linear map.
- Determine the eigenvalues and eigenvectors associated with an endomorphism and interpret them geometrically.
- Determine, classify and construct matrices of orthogonal transformations in the plane and in space.
- Apply vector calculus in the description of geometric objects.
- Classify conics and quadrics from its equation.

### Competences

#### Cross-disciplinary competences of the degree:

- EPS1. Capacity to solve problems and prepare and defence arguments inside the area of studies.
- EPS2. Capacity to gather and interpret relevant data, within the area of study, to judge and think about relevant subjects of social, scientific and ethical nature.
- EPS5. Capacity of abstraction and of critical, logical and mathematical thinking.
- EPS6. Capacity of analysis and synthesis.

#### Specific competences of the degree:

GEE1. Aptitude to apply the knowledge related with numerical and infinitesimal calculus, linear algebra, analytical and differential geometry, techniques and probabilistic methods and statistical analysis.

GEE2. Applied knowledge of the principles of general mechanics, the statics of structural systems, the geometry of masses, the principles and methods of analysis of the elastic behaviour of the solid.

## Subject contents

- 1. Matrices, determinants and systems of linear equations.
- 1.1 Types of matrices and matrix operations.
- 1.2 Inversible matrices.
- 1.3 Determinant of a square matrix of order n. Cases n = 2, n = 3 and n > 3.
- 1.4 Rank of a matrix.
- 1.5 Elementary transformations by rows. Gauss elimination method.
- 1.6 Expression of a system of linear equations by matrices.
- 1.7 Rouché-Frobenius theorem.
- 1.8 Cramer Systems

#### 2. Vector spaces.

- 2.1 Definition, properties and examples.
- 2.2 Linear combination: linear dependence and linear independence.
- 2.3 Generating system.
- 2.4 Basis of a vector space: definition, dimensions, components.
- 2.5 Vector Subspace. Linear varieties.
- 2.5.1 Description by using generators and equations.

- 2.5.2 Gauss Transformations. 2.5.3 Basis and dimension. 2.6 Basis change: matrix. 2.7 Operations on subspaces. 2.7.1 Containment and equality. 2.7.2 Intersection and sum. 2.7.3 Grassman formula. 2.7.4 Complementary and direct sum. 3. Linear transformations and endomorphism diagonalization. 3.1 Definition and properties of a linear transformation. 3.2 Determination by the image in a basis. 3.3 Kernel and image of a linear transformation. 3.4 Matrix representation of a linear transformation. 3.5 Changes of basis. Similar matrices. 3.6 Operations with linear applications. 3.6.1 Sum of applications. 3.6.2 Product by a scalar.
  - 3.6.3 Composition of applications.
  - 3.7 Diagonal matrix: eigenvalues and eigenvectors.
  - 3.8 Characteristic polynomial of an endomorphism: definition and calculation.
  - 3.9 Cayley-Hamilton theorem.
  - 3.10 Algebraic multiplicity.
  - 3.11 Subspace generated by an eigenvector: geometric multiplicity.
  - 3.12 Characterization of diagonalizable endomorphisms.

#### 4. Euclidean Geometry

- 4.1 Scalar product.
- 4.2 Norm induced by a scalar product.
- 4.3 Distances and angles.

- 4.4 Concepts of orthogonal and orthonormal.
- 4.5 Gram-Schmidt process of orthonormalization.
- 4.6 Gram's determinant. Volume of cuboid.
- 4.7 Vector Product.
- 4.8 Orthogonal transformations: in the plane and in space.
- 4.9 Isometries or rigid motions.

#### 5. Classification of conics and quadrics.

- 5.1 symmetric endomorphisms and scalar product.
- 5.2 symmetric bilinear form.
- 5.3 Range and signature: definition and calculation of these invariants.
- 5.4 Quadratic forms.
- 5.5 Quadratic varieties: canonical equation.
- 5.6 Classification of Euclidean conics and quadrics.
- 5.7 Other examples of curves. Trajectories. Parametrized curves.
- 5.8 Homothetic transformations.
- 5.9 Similarities.

## Methodology

Lectures: Classes to explain the theory and problem solving on the blackboard.

**Resolution of problems:** students solve problems individually during these sessions under the supervision of teachers of the subject.

**Exam:** two written tests are held during the semester. There is also a final retrieval test.

**Delivery of exercises:** Students deliver a weekly basis statement proposed by a teacher, who corrects and returns it to the student for his/her information.

## Development plan

Week	Methodology	Contents	Hours at classroom	Hours of autonomous work
Weeks 1 - 3	Lectures	Chapter 1	6	9
Weeks 1 - 3	Resolution of problems	Chapter 1	6	9

Weeks 4 - 6	Lectures	Chapter 2	6	9
Weeks 4 - 6	Resolution of problems	Chapter 2	6	9
Weeks 7 and 8	Lectures	Chapter 3 (sections 3.1 – 3.6)	4	6
Weeks 7 and 8	Resolution of problems	Chapter 3 (sections 3.1 – 3.6)	4	9
Week 9	1ª test of evaluation (exam)	Contents until the moment	1.5	
Weeks 10 - 12	Lectures	Chapter 3 (sections 3.7 -3.12)	6	9
Weeks 10 - 12	Resolution of problems	Chapter 3 (sections 3.7 -3.12)	6	9
Weeks 13 and 14	Lectures	Chapter 4	4	6
Weeks 13 and 14	Resolution of problems	Chapter 4	4	6
Week 15	Lectures	Chapter 5	2	3
Week 15	Resolution of problems	Chapter 5	2	6
Weeks 16 and 17	2ª test of evaluation (exam)	Contents since the first test and until the moment	1.5	
Week 20	Retrieval test	All the contents of the subject	2	

#### **Evaluation**

#### **Exam PA1**

Written exam (individual and compulsory)

Date: april and following the school calendar <a href="http://www.eps.udl.cat/ca/informacio-academica/horaris-i-calendaris/calendaris-dexamens/l">http://www.eps.udl.cat/ca/informacio-academica/horaris-i-calendaris-dexamens/l</a>

Percent on the final mark: 40%

Remark: To consider the mark of the given exercises a mark of at least 4 (over 10) in PA1 and PA2 is required.

#### Exam PA2

Written exam (individual and compulsory)

Date: june and following the school calendar <a href="http://www.eps.udl.cat/ca/informacio-academica/horaris-i-calendaris/calendaris-dexamens/l">http://www.eps.udl.cat/ca/informacio-academica/horaris-i-calendaris-dexamens/l</a>

Percent on the final mark: 40%

Remark: To consider the mark of the given exercises a mark of at least 4 (over 10) in PA1 and PA2 is required.

#### **Exercises**

Exercises to be corrected by the professor

Date: on each week of lectures there will be a statement to be solved.

Percent on the final remark: 20% (two of the given exercises are evaluated, each of them with a percent of 10% on the final mark)

Remark: To consider the mark of the given exercises a mark of at least 4 (over 10) in PA1 and PA2 is required.

#### Retrieval exam

Written exam (individual and non compulsory)

Date: june/july and following the school calendar <a href="http://www.eps.udl.cat/ca/informacio-academica/horaris-i-calendaris/calendaris-dexamens/">http://www.eps.udl.cat/ca/informacio-academica/horaris-i-calendaris/calendaris-dexamens/</a>

Percent of the final mark: 80%

Remarks: The content of this exam is all the topics of the subject.

To consider the mark of the given exercises a mark of at least 4 (over 10) in this resit exam is required.

All the students are allows to do this exam in order to improve their mark.

## **Bibliography**

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