

# DEGREE CURRICULUM LINEAR ALGEBRA

Academic year 2014-15

# Subject's general information

Subject name	Linear Algebra
Code	101401
Semester	2n S Continuous evaluation
Typology	Troncal
ECTS credits	6
Theoretical credits	0
Practical credits	0
Office and hour of attention	To concur Office 1.12 of the "Escola Politècnica Superior".
Department	Matemàtica
Modality	Presencial
Important information on data processing	Consult this link for more information.
Language	Catalan
Degree	Degree in Architectural Technology
Distribution of credits	See "Development plan".
Office and hour of attention	To concur Office 1.12 of the "Escola Politècnica Superior".
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# Subject's extra information

## Suggestions

Subject that requires continuous work throughout the semester in order to achieve its goals. It requires critical thinking and capacity for abstraction. You can find collections of the materials at the Cappont Campus (Building Aulari) and the Virtual Campus: http://cv.udl.cat

- Collection set of exercises with the numerical solutions.
- Resolutions of exams in previous years.

It is recommended to frequently visit the site at the Virtual Campus since all the information is announced there.

## The course as part of the academic plan

This course is in the 2nd semester of 1st year of teaching. It belongs to the module of "Basic Training", specifically in the field "Scientific Basis".

# Learning objectives

- Operate with matrices: sum, product, transposition and inverse.
- Perform elementary Gauss transformations in order to determine the rank of a matrix.
- Calculate determinants of square matrices of any order.
- Discuss and solve systems of linear equations.
- Solve systems using the method of Cramer.
- Determine if a vector is a linear combination of a set of vectors.
- Analyze whether a set of vectors is linearly dependent or not.
- Describe vector space and vector subspace.
- Make vector calculations in different bases. In particular, calculating the components of a vector on different bases.
- Calculating and applying the matrix shift base.
- Determine the dimension of a subspace.
- Do subspace operations: intersection, sum, direct sum.
- Apply the formula of Grassman.
- Determine a linear map from its image on a base.
- Calculate the kernel and image of a linear map.
- Relate explicit expression of a linear map with its matrix.
- Perform calculations in different linear basis. Apply matrix shift base on these calculations.
- Perform linear operations: addition, scalar product, composition.
- Determine the eigenvalues ??and eigenvectors associated with an endomorphism and interpret them geometrically.
- Use Cayley Hamilton theorem to study the characteristic polynomial of an endomorphism.
- Apply the usual scalar product of vectors to calculate distances and angles.
- Determine between orthogonal vectors.
- Apply the process of Gram-Schmidt orthomormalization to compute orthonormal bases
- Calculate volumes of parallelepipeds by Gram determinant.
- Determine, classify and construct matrices of orthogonal transformations in the plane and in space.
- Apply vector calculus in the description of geometric objects.
- Classify conics and quadrics from its equation.
- Calculate the invariants of a quadratic variety.
- Reason and contrast solutions.
- Synthesize the statement of a problem in order to express it mathematically.

- Use mathematical techniques to solve problems.
- Reason and analyze the results obtained from the calculation.

# Competences

EPS1. Capacity to solve problems and prepare and defence arguments inside the area of studies.

EPS2. Capacity to gather and interpret relevant data, within the area of study, to judge and think about relevant subjects of social, scientific and ethical nature.

EPS5. Capacity of abstraction and of critical, logical and mathematical thinking.

EPS6. Capacity of analysis and synthesis.

GEE1. Aptitude to apply the knowledge related with numerical and infinitesimal calculus, linear algebra, analytical and differential geometry, techniques and probabilistic methods and statistical analysis.

GEE2. Applied knowledge of the principles of general mechanics, the statics of structural systems, the geometry of masses, the principles and methods of analysis of the elastic behaviour of the solid.

# Subject contents

- 1. Matrices, determinants and systems of linear equations.
- 1.1 Types of matrices and matrix operations.
- 1.2 Inversible matrices.
- 1.3 Determinant of a square matrix of order n. Cases n = 2, n = 3 and n > 3.
- 1.4 Rank of a matrix.
- 1.5 Elementary transformations by rows. Gauss elimination method.
- 1.6 Expression of a system of linear equations by matrices.
- 1.7 Rouché-Frobenius theorem.
- 1.8 Cramer Systems

#### 2. Vector spaces.

2.1 Definition, properties and examples.

- 2.2 Linear combination: linear dependence and linear independence.2.3 Generating system.2.4 Basis of a vector space: definition, dimensions, components.
  - 2.5 Vector Subspace. Linear varieties.
  - 2.5.1 Description by using generators and equations.
  - 2.5.2 Gauss Transformations.
  - 2.5.3 Basis and dimension.
  - 2.6 Basis change: matrix.
  - 2.7 Operations on subspaces.
  - 2.7.1 Containment and equality.
  - 2.7.2 Intersection and sum.
  - 2.7.3 Grassman formula.
  - 2.7.4 Complementary and direct sum.

## 3. Linear transformations and endomorphism diagonalization.

- 3.1 Definition and properties of a linear transformation.
- 3.2 Determination by the image in a basis.
- 3.3 Kernel and image of a linear transformation.
- 3.4 Matrix representation of a linear transformation.
- 3.5 Changes of basis. Similar matrices.
- 3.6 Operations with linear applications.
- 3.6.1 Sum of applications.
- 3.6.2 Product by a scalar.
- 3.6.3 Composition of applications.
- 3.7 Diagonal matrix: eigenvalues and eigenvectors.
- 3.8 Characteristic polynomial of an endomorphism: definition and calculation.
- 3.9 Cayley-Hamilton theorem.
- 3.10 Algebraic multiplicity.
- 3.11 Subspace generated by an eigenvector: geometric multiplicity.

3.12 Characterization of diagonalizable endomorphisms.

## 4. Euclidean Geometry

- 4.1 Scalar product.
- 4.2 Norm induced by a scalar product.
- 4.3 Distances and angles.
- 4.4 Concepts of orthogonal and orthonormal.
- 4.5 Gram-Schmidt process of orthonormalization.
- 4.6 Gram's determinant. Volume of cuboid.
- 4.7 Vector Product.
- 4.8 Orthogonal transformations: in the plane and in space.
- 4.9 Isometries or rigid motions.

## 5. Classification of conics and quadrics.

- 5.1 symmetric endomorphisms and scalar product.
- 5.2 symmetric bilinear form.
- 5.3 Range and signature: definition and calculation of these invariants.
- 5.4 Quadratic forms.
- 5.5 Quadratic varieties: canonical equation.
- 5.6 Classification of Euclidean conics and quadrics.
- 5.7 Other examples of curves. Trajectories. Parametrized curves.
- 5.8 Homothetic transformations.
- 5.9 Similarities.

# Methodology

See "Development plan".

# Development plan

The topics described in the Contents part will be developed by <u>lectures</u> and exercices sessions during all the semester.

Hours at class: 28h Homework: 42h

The topics described in the Contents part will be developed by lectures and <u>exercices sessions</u> during all the semester.

Hours at class: 28h Homework: 42h

The evaluation, describen in the Evaluation part, will be developed by written exams and exercises.

Hours at class: 6h Homework: 10h

The exam revisions and the hours of attention are dedicated to solve issues and to clarify concepts.

Hours at class: 8h Homework: 0h

# **Evaluation**

#### **Exam PA1**

Written exam (individual and compulsory)

Date: april and following the school calendar

http://www.eps.udl.cat/info acad/horaris calendaris/calendari examens.html

Percent on the final mark: 40%

Remark: To consider the mark of the given exercises a mark of at least 4 (over 10) in PA1 and PA2 is required.

## Objectives

- Discuss and solve systems of linear equations.
- Analyze whether a set of vectors is linearly dependent or not.
- Describe vector space and vector subspace
- Apply the formula of Grassman.
- Reason and contrast solutions.
- Reason and analyze the results obtained from the calculation

## Exam PA2

Written exam (individual and compulsory)

Date: june and following the school calendar

http://www.eps.udl.cat/info acad/horaris calendaris/calendari examens.html

Percent on the final mark: 40%

Remark: To consider the mark of the given exercises a mark of at least 4 (over 10) in PA1 and PA2 is required.

## Exam PA2

## Objectives

- Determine the eigenvalues ??and eigenvectors associated with an endomorphism and interpret them geometrically
- Apply vector calculus in the description of geometric objects.
- Classify conics and quadrics from its equation
- Reason and contrast solutions.
- Reason and analyze the results obtained from the calculation.
- Synthesize the statement of a problem in order to express it mathematically
- Use mathematical techniques to solve problems.

#### **Exercises**

Exercises to be corrected by the professor

Date: on each week of lectures there will be a statement to be solved.

Percent on the final remark: 20% (two of the given exercises are evaluated, each of them with a percent of 10% on the final mark)

Remark: To consider the mark of the given exercises a mark of at least 4 (over 10) in PA1 and PA2 is required.

Objectives: All of the subject

#### Resit exam

Written exam (individual and non compulsory)

Date: june/july and following the school calendar

http://www.eps.udl.cat/info acad/horaris calendaris/calendari examens.html

Percent of the final mark: 80%

Remarks: The content of this exam is all the topics of the subject.

To consider the mark of the given exercises a mark of at least 4 (over 10) in this resit exam is required.

All the students are allows to do this exam in order to improve their mark.

Objectives: All of the subject

# **Bibliography**

#### **BASIC BIBLIOGRAPHY**

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- M. Alsina, J.M. Miret i A. Rio, Càlcul i Àlgebra. Resums i Problemes (amb solucions),1991.
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